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A MODEL OF INTERTEMPORAL COMPETITION FOR WATER LEVELS

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Abstract: Competition for water levels among different economic agents is a rather complex environmental economics problem. It is a game, because the agents are likely to act unilaterally. It is a dynamic game because water can be accumulated from season to season and also, the demand for water depends on the time of the year. The game is coupled through the state. We solve this game using a feedback Nash equilibrium concept. Our first solution does not satisfy environmental watchdog's expectations. Given an incentive scheme, we calculate another feedback Nash equilibrium that is more environmentally friendly than the previous one. The results are discussed in a general context of existence of feedback Nash equilibria in dynamic games.

Keywords: environmental management, feedback Nash equilibrium, Camargue water levels, diagonal strict concavity
JEL: C72 , O13
AMS: 91A05, 91A10, 91A25, 91A80

1. INTRODUCTION

The aim of this paper is twofold [1] . First, we want to solve a particular problem of intertemporal environmental management. The problem concerns fishermen and water-cress producers in the French region of Camargue. They compete for the water levels. In a stylised model, the fishermen utility is maximised by a high level of water, while the cress producers, at harvest times, prefer a low level of water. The agents can change the levels in a costly manner. We are interested in establishing, which water levels the agents can accept as unilaterally non-improvable. The Regional Government is a third player in this game. We want to examine an incentive scheme, utilised by the government and see whether it can compel the agents to agree upon environmentally friendly water levels.

The game at hand is *finite*-horizon (dynamic) with agents coupled through a state equation. Our second aim for this paper is to establish the conditions, under which the game can be solved for a *feedback Nash* (Markovian) equilibrium. This is a difficult problem which does not possess a general solution. Coupled dynamics' game models were analysed in [2] for the *infinite* horizon case. Finite horizon games with government subsidies and taxes with *open-loop* equilibria were studied in [8]. In [5], *feedback Nash* equilibria were analysed, however, for a non coupled-dynamics' case. We aim to contribute to that discussion

[1] This paper draws from and extends [7].

by proposing an approach to finite-horizon state-coupled games, which consists of solving stage games backwards in time.

The paper is organised as follows. In Section 2, a stylised two-agent game is presented to model the competition for water levels in Camargue. Then, in Section 3, we discuss a plausible solution concept for the problem at hand and also, the availability of a solution method for that concept. A stage-game feedback Nash equilibrium is suggested and computed in Section 4. This solution results environmentally *unfriendly*. A different solution, more pleasant to the regional government, is motivated and discussed in Section 5. Some concluding remarks are provided in Section 6.

2. COMPETITION FOR DIFFERENT WATER LEVELS

Camargue is a region in southern France where fishermen and water-cress producers (and hunters and more) compete for the water levels. Consider an annual cycle comprising K seasons. The level of water in season k is $x_k \geq 0$, $k = 1, \ldots K + 1$. The natural seasonal water level movements (*e.g.*, caused by evaporation and retention) are modelled through the parameter a_k ($a_k \geq 1$ or $0 < a_k < 1$). In the pristine (and deterministic) environment, the water levels would change as follows

$$\begin{aligned} x_{k+1} &= a_k x_k \qquad (1) \\ x_1 &- \text{given}, \quad k = 1 ... K. \end{aligned}$$

Assume there are $i = 1, \ldots N$ productive agents. In the computational part of the paper we will consider $N = 2$ representative players. They will be fishermen "F" and water-cress producers "P". However, in the theoretical sections, we will keep notation i and $-i$ to refer to the agent *at hand* and to the *other* agent(s), respectively. This will enable us to define some notions for any number of agents.

In each season, an agent may (costly) release, or let in, an amount of water $u_k^{(i)}$ ($u_k^{(i)} \geq 0 \quad \text{or} < 0$) to control the level x_{k+1} in the next season. So, the coupled dynamics' state equation for the water level in season $k + 1$ is

$$\begin{aligned} x_{k+1} &= a_k x_k + \sum_{i=1}^{N} u_k^{(i)} \qquad (2) \\ x_1 &- \text{given}, \quad k = 1 ... K. \end{aligned}$$

Suppose that the agents are earning an income that is composed of a fixed term and a of variable term where the latter depends on the water level. We will consider the variable part of the income only and call it *utility function*. For agent i, the utility function is as follows[2]

$$J_i(x_1, K; u^{(i)}, u^{(-i)}) = -\sum_{k=2}^{K+1} \left[\left(x_k - \bar{x}_k^{(i)} \right)^2 + q_k^{(i)} \left(u_{k-1}^{(i)} \right)^2 \right]. \qquad (3)$$

Notice that we have omitted a discount factor in the utility function. This means that the agents value each season's utility equally. This helps producing solutions that do not depreciate the future effects of the present actions. On the other hand, including a discount factor in (3) would not change the solution procedure.

In (3), symbols indexed $K + 1$ refer to the next year's first season. Variables $\bar{x}_k^{(i)}$ (with bars) represent preferred (given) water levels by agent i and may depend on a season ($k = 1, \ldots K + 1$). Parameter $q_k^{(i)}$ reflects a trade-off between the losses caused by a non desired water level and the level's control cost. The time dependence could capture the fact that some agents might value the level closeness in certain periods more than in some others. [3]

Hence the utility function (3) describes agents that are averse to large efforts $u_k^{(i)}$ and interested in keeping x_k close to $\bar{x}_k^{(i)}$. In real life, this could correspond to illegal (and punishable) opening of the region's sluices and to $\bar{x}_k^{(i)}$ being the constant income's maximisers.

Expression (3) would be maximised (zero) if the desired water levels were achieved and there would be no need for changing them. However, the desired levels are different for each player

$$\bar{x}_k^{(i)} \neq \bar{x}_k^{(-i)}$$

and it is impossible to achieve $J_i(x_1, K; u^{(i)}, u^{(-i)}) = 0$. This means that the agents are in conflict. We are interested to examine whether there are water levels, which the agents could accept as non improvable and what strategy would guarantee them.

3. A SOLUTION METHOD

We present a few definitions and theorems that help us to establish a solution to the above game.

[2] Thus defined utility is obviously negative. However, it would be positive if we allowed for a sufficiently large constant income. Adding it to (3) would not change equilibrium conditions.

[3] However, in the computational part of the paper, we assume that this parameter is constant through the year.

3.1 *A solution concept*

We are looking for a *feedback Nash equilibrium policy* (Markovian) $\{u^{(i)}(x_k)\}, k = 1 \dots K, i = 1 \dots N$ defined as

$$\{u^{(i)}(x_k)\}_{k=1\dots K} = \tag{4}$$
$$\operatorname{arg\,equil}\{J_i(x_1, K; (\cdot), (\cdot)), J_{-i}(x_1, K; (\cdot), (\cdot))\}.$$

A policy of that kind would be based on available observation x_k, maximise the agent's utility function (3) and be unilaterally non improvable. There are no guarantees that such an equilibrium exists.

3.2 *Sufficient and uniqueness conditions*

We know that, a *concave game i.e.*, such that each player's utility function is continuous in all players' actions and concave [4] with respect to his own strategy while the other players' strategies remain fixed, must have at least one Nash equilibrium. [5]

However, for the environmental game described in Section 2, we want to establish not only an equilibrium existence but also its uniqueness. The need for the solution uniqueness is typical of environmental management problems. The Regional Government needs to know what the equilibrium is. For it were many, and some of them less environmentally friendly than some others, the government would not know, which protective measures, if any, to take.

The seminal Rosen's paper [9] relates conditions for equilibrium uniqueness of a *concave* game solution

$$\hat{\mathbf{u}}_i = \operatorname{arg\,equil}\left\{f^{(i)}(\mathbf{u}), f^{(-i)}(\mathbf{u})\right\}, \tag{5}$$

where $f^{(i)}(\mathbf{u})$ is player i's utility function and $\mathbf{u}$ is the combined policy vector

$$\mathbf{u} = \begin{bmatrix} u_i \\ u_{-i} \end{bmatrix} \in \mathbb{X} \subset \mathcal{R}^m$$

(set $\mathbb{X}$ is compact and $m \geq N$ denotes the total number of all players' actions), to the concept of *diagonal strict concavity* (DSC) of the *joint payoff function.* We will define and explain those notions.

The *joint payoff function* is defined as [6]

$$\phi(\mathbf{u}) \equiv \sum_{i=1}^{n} f^{(i)}(\mathbf{u}). \tag{6}$$

In broad terms, a game whose joint payoff is DSC or, for shortness, a game which is DSC, is one in which each player has more control over his payoff than the other players have over it. This is a rather common, and desired, feature of many economic models. The formal definition of DSC is as follows.

Definition 1. The game is called diagonally strictly concave (DSC) for $\mathbf{u} \in X$ if for every $\mathbf{u}^0, \mathbf{u}^1 \in X$ we have

$$(\mathbf{u}^1 - \mathbf{u}^0)' g(\mathbf{u}^0) + (\mathbf{u}^0 - \mathbf{u}^1)' g(\mathbf{u}^1) > 0 \tag{7}$$

where $'$ denotes transposition, $g(\mathbf{u})$ is the pseudo-gradient of $\phi(\mathbf{u})$

$$g(\mathbf{u}) \equiv \begin{bmatrix} \dfrac{\partial f_1(\mathbf{u})}{\partial \mathbf{u}_1} \\ \vdots \\ \dfrac{\partial f_N(\mathbf{u})}{\partial \mathbf{u}_N} \end{bmatrix}, \tag{8}$$

where $\mathbf{u}_j, j = 1 \dots N$ is the part of $\mathbf{u}$, which contains actions controlled by player j.

If utility functions are twice differentiable, a criterion for DSC is simple and consists of checking whether the pseudo-Hessian of $\phi(\mathbf{u})$

$$\mathcal{H} = H + H' \tag{9}$$

where H is the Jacobian of $g(\mathbf{u})$

$$H = \begin{bmatrix} \dfrac{\partial^2 f_1(\mathbf{u})}{\partial \mathbf{u}_1^2} & \dfrac{\partial^2 f_1(\mathbf{u})}{\partial \mathbf{u}_2 \partial \mathbf{u}_1} & \cdots & \dfrac{\partial^2 f_1(\mathbf{u})}{\partial \mathbf{u}_N \partial \mathbf{u}_1} \\ \dfrac{\partial^2 f_2(\mathbf{u})}{\partial \mathbf{u}_1 \partial \mathbf{u}_2} & \dfrac{\partial^2 f_2(\mathbf{u})}{\partial \mathbf{u}_2^2} & \cdots & \dfrac{\partial^2 f_2(\mathbf{u})}{\partial \mathbf{u}_N \partial \mathbf{u}_2} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{\partial^2 f_N(\mathbf{u})}{\partial \mathbf{u}_1 \partial \mathbf{u}_N} & \dfrac{\partial^2 f_N(\mathbf{u})}{\partial \mathbf{u}_2 \partial \mathbf{u}_N} & \cdots & \dfrac{\partial^2 f_N(\mathbf{u})}{\partial \mathbf{u}_N^2} \end{bmatrix} \tag{10}$$

is negative definite. This will be true for games, in which the utility function concavity in the agents's own policy cannot be "destroyed" by the opponents' actions.

As the following theorem specifies it, confirming pseudo-Hessian's negative definiteness is sufficient for uniqueness of a Nash equilibrium (see [9] or [4]).

Theorem 1. In a game $\operatorname{equil}\{f^{(i)}(\mathbf{u}), f^{(-i)}(\mathbf{u})\}$ if the joint payoff function $\phi(\mathbf{u})$ is DSC, then there exists a unique Nash equilibrium.

[4] These assumptions can be weaken, see *e.g.*, [3].
[5] Notice that for our dynamic game, checking the payoff concavity assumption might be non trivial as the utility function is a sum and depends on the coupling equation (2). We will address this problem by examining stage games in Section 4.2.
[6] Rosen's definition is more general and allows for Regional Government "appraisal" of individual agent utilities, however, we do not need the appraisal in this paper.

4. A FEEDBACK NASH EQUILIBRIUM

4.1 *The method*

We know from Section 3.2 how to determine whether a game has a unique equilibrium. However, our game (4) is in J_i, J_{-i} (and subject to a dynamic coupling equation (2)) and not in $f^{(i)}, f^{(-i)}$ which are "just" concave functions (see (5)). To solve our dynamic game, we will combine the existence and uniqueness theorem (Theorem 1) with the Bellman optimality principle. This means, in broad terms, that we will examine uniqueness of *stage games* for each stage $s = K, K-1, \ldots 1$ (backward in time). And, at each stage, the role of utilities $f^{(i)}$ will play the *utility-to-go* functions, defined below as $F^{(i)}(x_s, s; \cdot, \cdot)$.

4.2 *Stage games*

Define $V_s^{(i)}(x_s, s)$, a stage *optimal value function* for player i as

$$V_s^{(i)}(x_s, s) = \max_{u_s^{(i)}} F^{(i)}(x_s, s; u_s^{(i)}, u_s^{(-i)}(x_s)) \quad (11)$$

$$s = K, ...1 \qquad \text{where}$$

$$F(x_s, s; u_s^{(i)}, u_s^{(-i)}(x_s)) \equiv \quad (12)$$
$$\left\{ - \left[\left(a_s x_s + u_s^{(i)} + u_s^{(-i)}(x_s) - \bar{x}_s^{(i)} \right)^2 + q_s^{(i)} \left(u_s^{(i)} \right)^2 \right] + V_{s+1}^{(i)}(x_{s+1}, s+1) \right\},$$

$$V_{K+1}^{(i)}(x_{K+1}, K+1) = \max_{u^{(i)}} - \left(x_{K+1} - \bar{x}_{K+1}^{(i)} \right)^2 \quad (13)$$

The following theorem [7] establishes a basis for using dynamic programming as a computational technique for feedback Nash equilibria (subgame perfect) in dynamic games.

Theorem 2. If there exist the value functions $V_s^{(i)}(x_s, s)$ and the strategies $u_s^{(i)}(x_s)$, which satisfy equations (11), (12) for $s = K, K-1, \ldots 1$, $x \in \mathbb{X}$, then the strategy pair

$$u = (u^{(i)}, u^{(-i)}), \ \ u^{(i)} = \{u_s^{(i)} : \ s = K, K-1, \ldots 1\}$$

constitutes a feedback Nash equilibrium of the dynamic game with the feedback information pattern [8]. Moreover, the value functions $V_s^{(i)}(x_s, s)$ represent the optimal utility of player i for the game starting at (x_s, s). In particular,

$$V_0^{(i)}(x_0, 0) = \max J_i(x_1, K; (\cdot), (\cdot)). \quad (14)$$

First, notice that each stage game is a concave game. Indeed, it is easy to see that each stage value function $F^{(i)}(x_s, s; u_s^{(i)}, u_s^{(-i)}(x_s))$ is a (negative) quadratic function of $u^{(i)}$ and continuous in $u^{(-i)}$. If so, it makes sense to ask whether the stage games have unique equilibria.

We will establish uniqueness of stage equilibria using, at each stage for each $s = K, ...1$, first, Theorem 1 to see if the equilibrium is unique and, then, Theorem 2 to compute the equilibrium strategy. Indeed, if a unique equilibrium

$$\{u^{(i)}(x_s)\} = \operatorname{arg\,equil} \Big\{ F^{(i)}(x_s, s; u_s^{(i)}, u_s^{(-i)}(x_s)), F^{(-i)}(x_s, s; u_s^{(-i)}, u_s^{(i)}(x_s)) \Big\}. \quad (15)$$

exists for each $s = K, ...1$ then, by construction, the unique, feedback Nash equilibrium (4) also exists.

The uniqueness theorem (Theorem 1) says that if the game's *joint payoff function* is *diagonally strictly concave* (DSC) then the equilibrium is unique. So, we will construct

$$\Phi(x_s, s; u_s^{(i)}, u_s^{(-i)}) \equiv \quad (16)$$
$$F(x_s, s; u_s^{(i)}, u_s^{(-i)}(x_s)) + F(x_s, s; u_s^{(i)}(x_s), u_s^{(-i)}),$$

for each $s = K, ...1$ and then, through checking the pseudo-Hessian of Φ will establish uniqueness of (15) and then of (4).

4.3 *Model coefficients*

For a horizon of length $K = 4$ (four quaters, say), the closed form formulae for stage games' joint payoffs, pseudo-Hessians, strategies *etc.*, are obtainable through MATLAB SYMBOLIC MATHS or MATHEMATICA. However, they are too long and complicated to analyse them as analytical expressions. We will assume certain plausible model parameter values, or ranges for some of them, and analyse the solutions numerically.

Evaporation and retention parameters a_k. We assume from now onwards that $k = 1$ corresponds to winter. Consequently, $k = 2, 3, 4$ denote spring, summer and autumn, respectively. In Camargue, in relative terms, spring and autumn are wet, summer is dry and winter is "neutral". We assume the following values for vector a:

$$a = \begin{bmatrix} \frac{\sqrt{5}}{2}, & \frac{4}{5}, & \frac{\sqrt{5}}{2}, & 1.0000 \end{bmatrix}. \quad (17)$$

This means that, if there were no human interventions, there would on average be 12% more

[7] "Standard" in dynamic games, see [1], Theorem 6.6, pp. 284-285.

[8] Such an equilibrium is subgame perfect and often called Markovian.

water in spring than in winter, 20% less in summer than in spring and 12% more in autumn than in summer; finally the amount of water carried from autumn to winter would be the same.

Preference level parameters $\bar{x}_k^{(i)}$. As said, water-cress producers prefer lower water levels during certain times (because of the growth or harvest requirements). We index the cress producers P (and use F for the fishermen) and model the level preference parameters $\bar{x}_k^{(i)}$ for all players as follows:

$$\bar{x}_k^F = [\bar{x}_1,\, \bar{x}_2,\, \bar{x}_3,\, \bar{x}_4,\, \bar{x}_1,\,] \tag{18}$$
$$\bar{x}_k^P = [\bar{x}_1,\, \beta\bar{x}_2,\, \bar{x}_3,\, \beta\bar{x}_4,\, \bar{x}_1,\,]\,. \tag{19}$$

In this study, we will assume that fishermen like to have "1.2" all the time ($\bar{x}_k^F = 1.2, \quad \forall k$). Regarding the cress producers, we suppose that the critical seasons for them are spring and autumn and that they prefer 75% of the water level the fishermen want. So, $\beta = .75$.

Cost coefficients. The next assumption concerns the cost coefficients $q_k^{(i)}$. We assume that the cost of changing the water level is constant and identical for each player *i.e.*, $q_k^{(i)} = q_k^{(-i)} = q > 0$.

4.4 *A numerical solution*

We first check existence and uniqueness of stage games' strategies (15). The following Figure 1 shows definiteness of the pseudo-Hessian of $-\Phi\left(x_s, s; u_s^{(i)}, u_s^{(-i)}\right)$ [9] . This is a symmetric 2×2 matrix. Sufficient for its positive definiteness are: positive (both) entry "1,1" and the determinant. The left panels show the "1,1" entries; the right panels show the determinants; all as functions of the cost coefficient q. The most upper panels are drawn for the last stage game. The next ones are for the last two-stage game. *Etc.* We observe that the requested strict positive definiteness is guaranteed, however, it depends on the cost coefficient q and worsens for smaller q. This is not surprising because for the no-cost case ($q = 0$), the players could use any control and an equilibrium would never exist. We also notice that definiteness improves slightly for shorter horizons.

We know that if the equilibrium (15) is unique then we can compute it by solving (11) simultaneously for $i = 1, 2$.

The computed equilibrium strategy realisations and the corresponding state evolution are presented in Figure 2 (upper and bottom panels,

[9] The negative pseudo-Hessian needs to be positive definite.

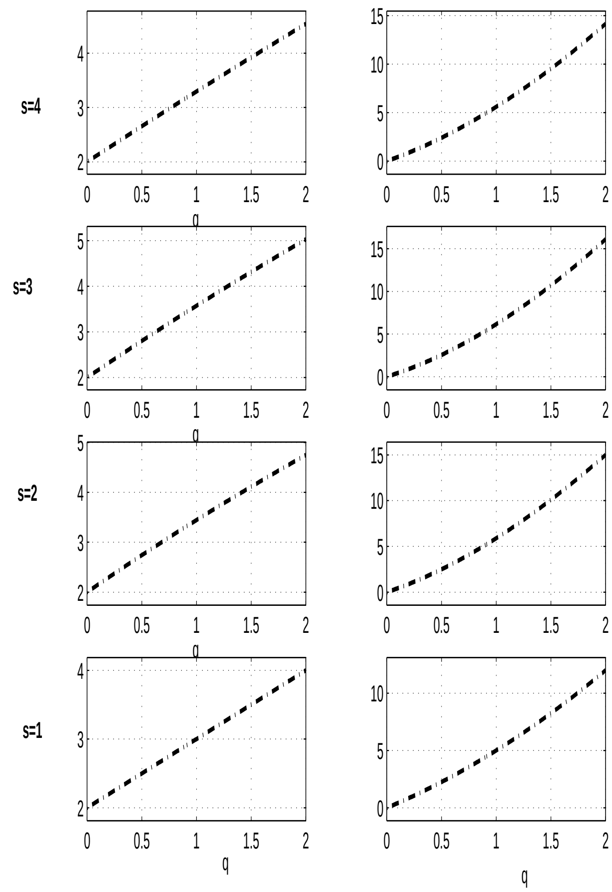

Fig. 1. Pseudo-Hessian definiteness.

respectively), for two values of the control cost parameter q (and $\bar{x}_k^F = 1.2, \quad \forall k,\ \beta = .75,\ x_1 = 1$ and the natural level fluctuations governed by (17)). The positive bars on the strategy graph (upper panel) are for fishermen and the negative ones for cress-producers. The dashed lines correspond to $q = .5$ and the dash-dotted ones to a "cheaper" control characterised by $q = .1$. The natural water level fluctuations are shown as the thin solid line in the bottom panel. The fishermen's preference level is 1.2 $\forall k$ while the cress producers' differs for $k = 2$ and $k = 4$ and equals .9 . For clarity of the figure, these levels are here omitted; they will be sketched later in Figures 4 and 6.

Examination of the figure panels makes it is obvious to conclude that sustaining the natural water level $x_1 = x_5 = 1$ is impossible, especially for cheap controls (or lax penalties for tempering with the sluices). Therefore a need for a government intervention becomes a necessity to lessen the environmental impact of agents' economic activities. In the next section, we examine whether an intervention of the Regional Government is likely to stabilise the water level around 1 and how much this may cost.

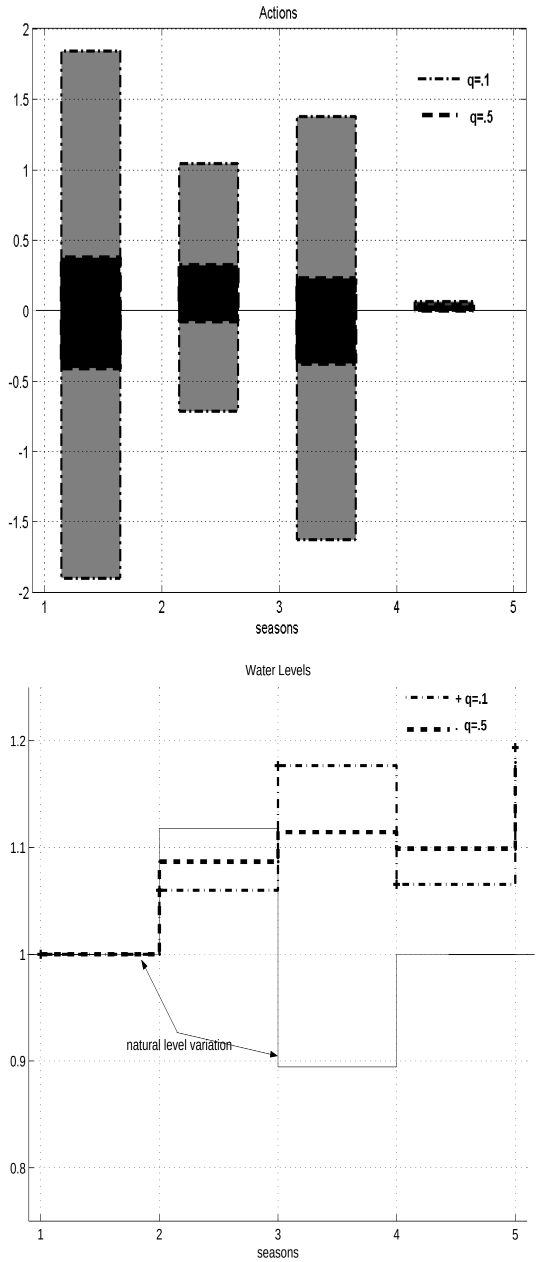

Fig. 2. Equilibrium strategy and state realisations.

5. A MODIFIED DYNAMIC GAME

5.1 *A one-year problem*

Certainly, a Regional Government will be concerned if x_5 differs substantially from x_1. We will now examine how adding an incentive (or penalty) term to a player's utility function can modify the players' behaviour. In particular, we want to show that the government can control the agent to an *environmentally friendlier* equilibrium through the use of an incentive scheme.

In [6], a leader controlled *satisfactorily* a water level through use of certain correction parameters, in the context of an optimal control problem. We will follow that approach and apply it here to improve a dynamic game's outcome.

Consider the following scheme: if, after a year (*i.e.*, K periods), the levels x_{K+1} and x_1 are "close" to each other (in the sense of an environmental watch dog's standards), each player will be paid a bonus

$$(W - (x_{K+1} - x_1)^2)U, \qquad U, W \geq 0. \qquad (20)$$

If the difference between the levels is large, the bonus will become a penalty. So, now we will look for an equilibrium where the players maximise the following utility functions

$$I_i(x_1, K; u^{(i)}, u^{(-i)}) = \left[W - (x_{K+1} - x_1)^2\right] U \\ - \sum_{k=2}^{K+1} \left[\left(x_k - \bar{x}_k^{(i)}\right)^2 + q_k^{(i)} \left(u_{k-1}^{(i)}\right)^2 \right]. \qquad (21)$$

We have first to examine existence of stage equilibria defined analogously to (21) where the functions J_i are replaced by I_i.

The following Figure 3 is analogous to Figure 1 and shows definiteness of the stage games' pseudo-Hessians for varying values of the incentive parameter U.

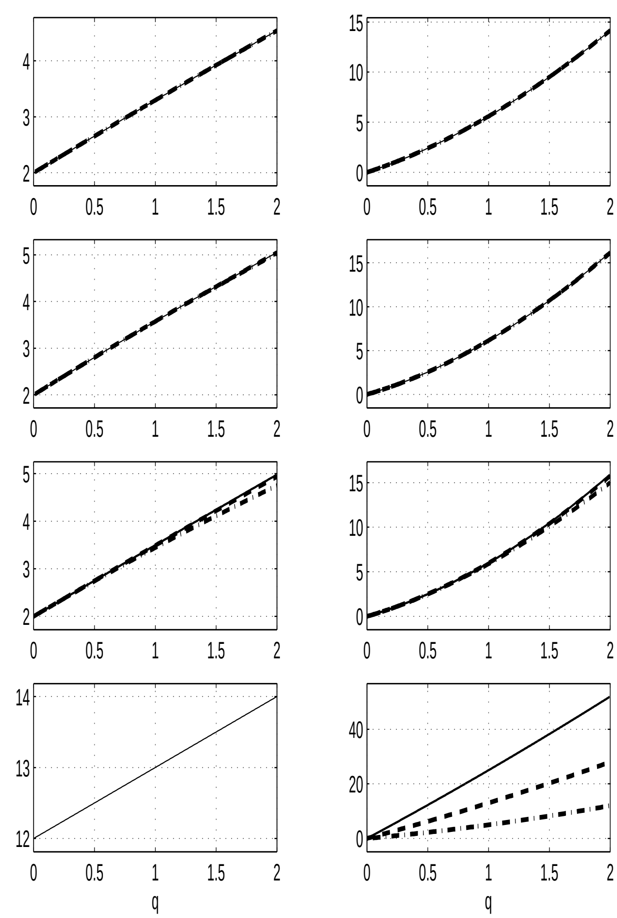

Fig. 3. Pseudo-Hessian definiteness for varying incentive levels.

As before, the left panels represent entry "1,1" values and the right panels show the pseudo-Hessian determinants. The dash-dotted lines correspond to $U = 0$ and are identical with those in Figure 1. The dash lines correspond to $U = 2$. The solid lines are drawn for $U = 5$. It is obvious from the figure that unique equilibria exist for $U \geq 0$. For $s = 1$, larger incentives clearly improve the (negative) pseudo-Hessian positive definiteness. This is an encouraging result, which suggests that the regulator will have a range of U values, for which equilibria exist and that s/he will be able to select an environmentally friendly solution.

Indeed, Figure 4 shows (for $q = .5$) that the modified water levels are such that x_5 is much closer to x_1 than before. The dash-dotted line corresponds to $U = 0$ and is considered *environmentally unfriendly* (exactly as in Figure 2). The dash line ($U = 2$) and the solid line ($U = 5$) show that the new equilibrium strategy can bring x_5 very close to x_1.

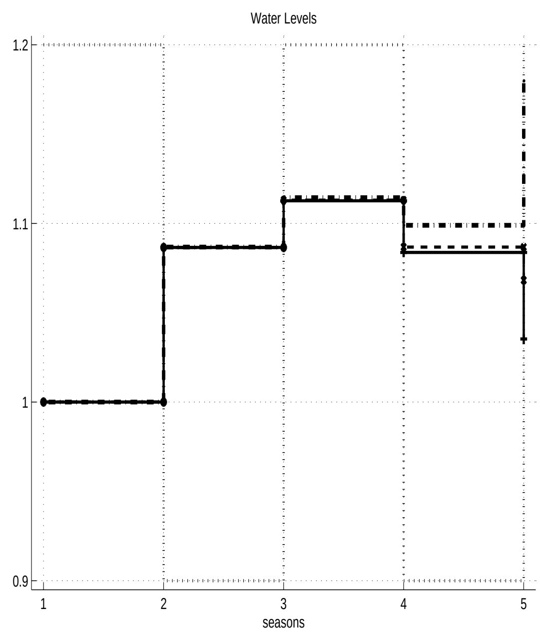

Fig. 4. Modified state realisations.

The equilibrium actions leading to the above water levels are shown in Figure 5 upper and bottom panels, for the fishermen (Player 1) and cress producers (Player 2), respectively.

We notice that the players behave "rationally" in that they react to the government control parameter U by modifying mostly their last period's actions. This seems sufficient to fulfill the government's aim to bring the *final* water level close to x_1.

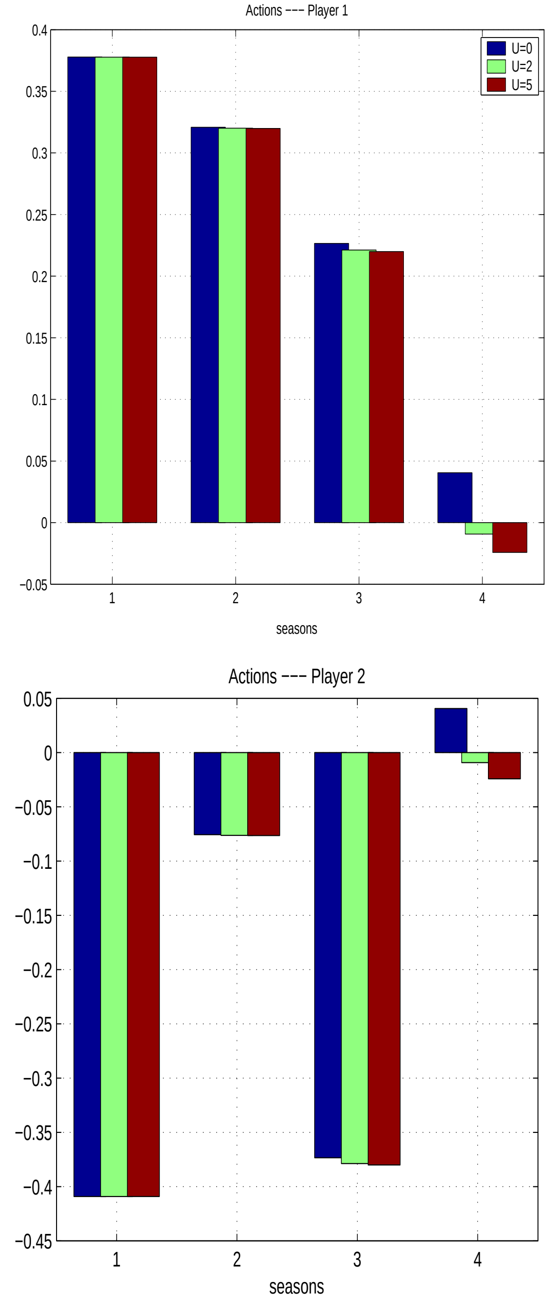

Fig. 5. Modified equilibrium strategy realisations.

5.2 *A multi-year extension*

An obvious question to ask is whether the above dynamic 4-season equilibria will be maintained in the second and subsequent years. To answer this question we have computed the equilibrium strategies for $U = 0$ and $U = 5$ ($q = .5$) for the second year's initial states equal to x_5. Figure 6 shows the corresponding water levels. The dash dotted line corresponds to $U = 0$ and the solid line to $U = 5$. It is apparent that the equilibrium strategies (aided or not by the incentive scheme) are such that "winter" equilibrium levels remain largely unchanged.

The dash line (mostly overlapping with the other lines) shows the water levels in a "transitory" period *i.e.*, second year. After $U = 0$ ("applied" in the first year), the incentive scheme with $U = 5$ was implemented in the second year (from $x_5 = 1.1798$). The government incentive appears efficient in that it motivates agents to such actions that bring the next winter's level close to the desired one. This means that the incentive scheme can successfully control the "next" winter's level for a broad range of initial conditions. This speaks well about the scheme robustness *i.e.*, that it should work satisfactorily in the presence of a stochastic noise or other uncertainties.

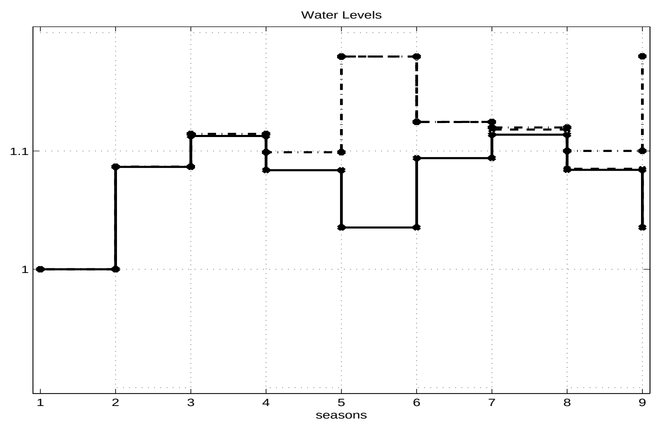

Fig. 6. Two-year water levels.

6. CONCLUDING REMARKS

The first aim of this paper was to solve a water level competition game in Camargue. The obtained results tell us that it is possible for a Regional Government to compel agents to a feedback Nash equilibrium where environmental standards are obeyed. The sufficient features of the model that provides the desired equilibrium include quadratic action costs and a quadratic bonus-penalty function (see (20). The former corresponds to the "natural" way of modelling getting away with small infraction (little water let in or out) and being caught and punished for large amounts of water transfer. The latter is a matter of choice for the Regional Government. These are realistic model requirements hence we should expect sensible results from our model provided that its parameters were properly calibrated.

Our second aim of this paper was to contribute to methodology of solutions of state-coupled dynamic games. We have demonstrated that checking stage games for DSC and solving them recursively in backward time leads to the establishment of feedback Nash equilibria.

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